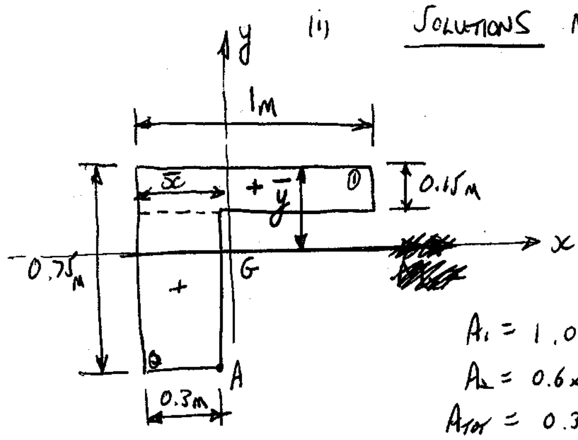
(1) SOLUTIONS MM2MS2
0910

Q1



$A_1 = 1 . 0.15 = 0.15 \, m^2$
$A_2 = 0.6 \times 0.3 = 0.18 \, m^2$
$A_{TOT} = 0.33 \, m^2$

Centroid

$$0.33 . \bar{y} = 0.15 . 0.075 + 0.18 . 0.45$$

$$\bar{y} = \underline{0.2795 \, m} \text{ from top}$$

$$0.33 . \bar{x} = 0.15 . 0.5 + 0.18 . 0.15$$

$$\bar{x} = 0.309 \, m \text{ from } \cancel{\text{top}} \text{ left side} \quad ? \qquad [5 \text{ marks}]$$

Principle 2nd Moments

$$I_x = \frac{1 . 0.15^3}{12} + 0.15 . 0.2045^2 + \frac{0.3 . 0.6^3}{12} + 0.18 . 0.1705^2$$

$$= 2.813 . 10^{-4} + 62.73 . 10^{-4} + 54 . 10^{-4} + 52.33 . 10^{-4}$$

$$= 171.87 . 10^{-4}$$
$$= \underline{\underline{0.0172}} \, m^4$$

$$I_y = \frac{0.15 . 1^3}{12} + 0.15 . 0.191^2 + \frac{0.6 . 0.3^3}{12} + 0.18 . 0.159^2$$
$$= 0.0125 + 5.472 . 10^{-3} + 1.35 . 10^{-3} + 4.55 . 10^{-3}$$
$$= \underline{\underline{0.0239}} \, m^4$$

Q1 (contd)

$I_{xy} = 0.15 \times 0.191 \times 0.2045 + 0.18 \times 0.159 \cdot (-0.1705)$

$= 5.859 \cdot 10^{-3} + 4.8797 \cdot 10^{-3}$

$= \underline{0.0107\ m^4}$

Mohr's Circle

Centre $C = \frac{I_{xx}+I_{yy}}{2} = \underline{0.02055}\ m^4$

Radius $R = \sqrt{\left(\frac{I_{xx}-I_{yy}}{2}\right)^2 + I_{xy}^2} = \sqrt{1.122 \cdot 10^{-5} + 1.145 \cdot 10^{-4}}$

$= \underline{0.0112\ m^4}$

$I_p = C + R = \underline{0.03175\ m^4}$

$I_q = C - R = \underline{0.00935\ m^4}$

[10 marks] inc. Mohr's circle sketch

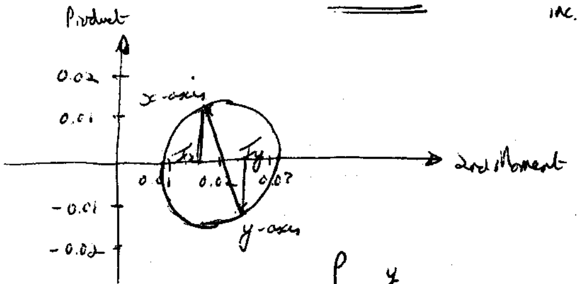
$\sin 2\theta = \frac{I_{xy}}{R} = \frac{0.0107}{0.0112} = 0.9554$

$2\theta = 72.82$

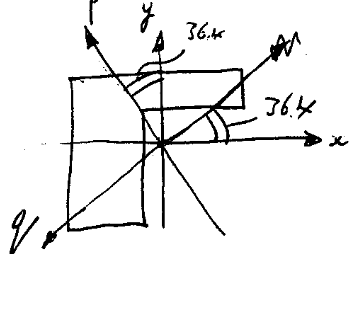
$\theta = \underline{36.4^\circ}$

[5 marks] inc. sketch

Mohr's Circle Sketch



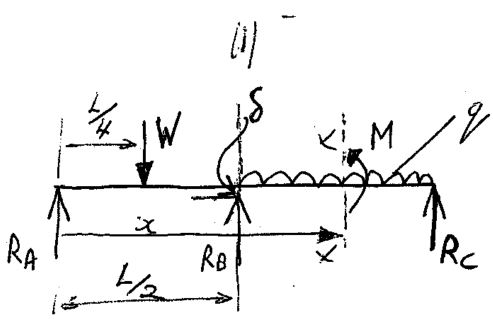
Sketch of Principal Axes



(i)

MM2MS2
0910

Q2



Moments about $\quad M - R_A x + W\langle x-\tfrac{L}{4}\rangle - R_B\langle x - \tfrac{L}{2}\rangle + \frac{q}{2}\langle x-\tfrac{L}{2}\rangle^2 = 0$

$$EI\frac{d^2y}{dx^2} = -M = -R_A x + W\langle x-\tfrac{L}{4}\rangle - R_B\langle x-\tfrac{L}{2}\rangle + \frac{q}{2}\langle x-\tfrac{L}{2}\rangle^2$$

$$EI\frac{dy}{dx} = -\frac{R_A x^2}{2} + \frac{W\langle x-\tfrac{L}{4}\rangle^2}{2} - \frac{R_B\langle x-\tfrac{L}{2}\rangle^2}{2} + \frac{q}{6}\langle x-\tfrac{L}{2}\rangle^3 + A$$

(5 marks) $$EIy = -\frac{R_A x^3}{6} + \frac{W\langle x-\tfrac{L}{4}\rangle^3}{6} - \frac{R_B\langle x-\tfrac{L}{2}\rangle^3}{6} + \frac{q}{24}\langle x-\tfrac{L}{2}\rangle^4 + Ax + B$$

when $x=0$ $y=0$ $\therefore$ $B=0$

when $x=L$ $y=0$ $\therefore$ $0 = -\dfrac{R_A L^3}{6} + \dfrac{W\langle\tfrac{3L}{4}\rangle^3}{6} - \dfrac{R_B\langle\tfrac{L}{2}\rangle^3}{6} + \dfrac{q}{24}\langle\tfrac{L}{2}\rangle^4 + AL$ ①

(5 marks) when $x=\frac{L}{2}$ $y=\delta$ $\therefore$ $EI\delta = -\dfrac{R_A L^3}{48} + \dfrac{W\langle\tfrac{L}{4}\rangle^3}{6} + \dfrac{AL}{2}$ ②

Eq ① $- 2\times$ Eq ② $\Rightarrow$

$$-2EI\delta = \frac{-R_A L^3}{6} + \frac{R_A L^3}{24} + \frac{W\langle\tfrac{3L}{4}\rangle^3}{6} - \frac{W\langle\tfrac{L}{4}\rangle^3}{3} - \frac{R_B\langle\tfrac{L}{2}\rangle^3}{6} + \frac{q}{24}\langle\tfrac{L}{2}\rangle^4 \quad ③$$

Equilibrium

Moments about C $\quad R_A L + R_B\dfrac{L}{2} = W.\dfrac{3L}{4} + q\dfrac{L}{2}.\dfrac{L}{4}$ ④

Q2
(cont)

$W = 60\,kN \quad L = 6\,m$

$q = 20\,kN/m \quad EI = 4.10^7\ Nm^2$

$\delta = 6\,mm$

$④ \Rightarrow \quad 6R_A + 3R_B = 60.10^3 \cdot \frac{3}{4} \cdot 6 + 20.10^3 \cdot \frac{6^2}{8} = \underline{3.6.10^5} \quad ⑦$

$③ \Rightarrow \quad -2.4.10^7.6.10^{-3} = R_A(9-36) - 4.5R_B + 9.113.10^5 - 0.675.10^5 + 0.675.10^5$

$\underline{1.391.10^6} = 27R_A + 4.5R_B \quad ⑥$

$⑥ - 1.5 \times ⑦$

$\Rightarrow \quad 18R_A = 13.91.10^5 - 5.4.10^5 = 8.51.10^5$

$\therefore R_A = \underline{47.28\,kN}$

From ⑦ $\quad R_B = \underline{25.44\,kN}$

5 marks

Equilibrium of vertical forces

$R_A + R_B + R_C = 60\,kN + 60\,kN = 120\,kN$

$\therefore R_C = \underline{47.28\,kN}$ [15 marks]

Deflection at load

Use ② to find A $\quad \frac{A.L}{2} = EI\delta + \frac{R_A L^3}{48} - \frac{W\left(\frac{L}{4}\right)^3}{6}$

$3A = 1.391.10^6 + \frac{47.28.10^3 . 6^3}{48} - \frac{60.10^3(1.5)^3}{6}$

$= 1.391.10^6 + 0.213.10^6 - 0.0338.10^6$

$\therefore A = \underline{0.523.10^6}$

Q2 (cont)

From deflection equation, at W,

$$EIy = -\frac{R_A\left(\frac{L}{4}\right)^3}{6} + A.\frac{L}{4}$$

$$EIy = -2.66.10^4 + 78.45.10^4$$

7.579 [redacted] $.10^5$ / 4.10^7 = 18.9 [redacted] mm

[5 marks]

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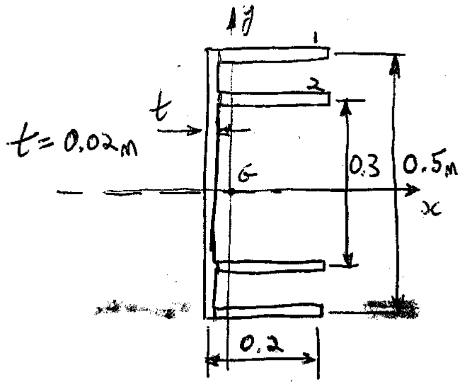
Q3

(b)



<u>2nd Moment of Area</u>

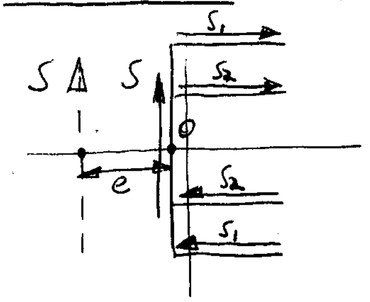
$$I_{WEB} = \frac{0.02 \cdot 0.52^3}{12} = \underline{2.343 \cdot 10^{-4}} \, m^4$$

$$I_{FLANGE1} = \frac{0.19 \cdot 0.02^3}{12} + 0.19 \cdot 0.02 \cdot 0.25^2$$
$$= 1.267 \cdot 10^{-7} + 2.375 \cdot 10^{-4}$$
$$= \underline{2.376 \cdot 10^{-4}} \, m^4$$

$$I_{FLANGE2} = 1.267 \cdot 10^{-7} + 0.19 \cdot 0.02 \cdot 0.15^2$$
$$= 1.267 \cdot 10^{-7} + 8.55 \cdot 10^{-5}$$
$$= \underline{0.856 \cdot 10^{-4}} \, m^4$$

7 $I_{TOT} = I_{WEB} + 2(I_{FLANGE1} + I_{FLANGE2})$ [7 marks]
$$= \underline{8.807 \cdot 10^{-4} \, m^4}$$

<u>SHEAR CENTRE</u> – this is on the axis of symmetry, the x-axis
Taking Moments about O



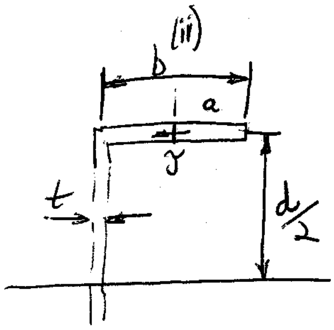
2 $S.e = 2S_1 \, 0.25 + 2S_2 \, 0.15$

$Se = 0.5S_1 + 0.3S_2$

~~[illegible]~~ 1kN ~~[illegible]~~

Q3 (cont) Shear Stresses in a flange

(ii)



$$\tau = \frac{S}{I_z} A\bar{y} = \frac{S}{It}(at).\frac{d}{2} = \frac{S.a.d.}{2I}$$

Shear force in a flange $\quad S_1 = \int_0^b \tau t\, da = \int_0^b \frac{S a d}{2I} t\, da$

5 $\quad = \underline{\dfrac{S d t b^2}{4I}}$

Top & Bottom flanges

$d = 0.5 \quad t = 0.02 \quad b = 0.2$

$I = 8.807 \cdot 10^{-4}$

$S_1 = S \cdot \dfrac{0.5 \cdot 0.02 \cdot 0.2^2}{4 \cdot 8.807 \cdot 10^{-4}}$

$= \underline{0.1135\, S}$

4

Intermediate flanges

$d = 0.3 \quad t, b, I$ as above

$S_2 = \underline{0.0681\, S}$

SHEAR CENTRE

$$S.e = 0.5\, S_1 + 0.3\, S_2 = 0.5 \cdot 0.1135\, S + 0.3 \cdot 0.0681\, S$$

2 $\quad \underline{\underline{e = 0.077}}$ m

[13 marks]

(1)

MM2MS2
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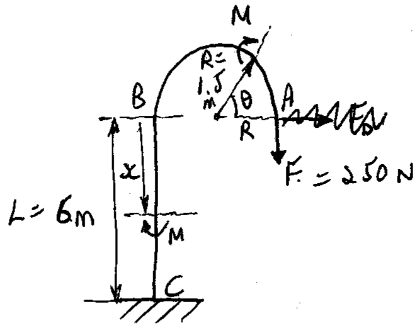
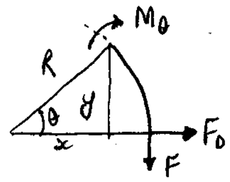
Q4



R = 1.5m
L = 6m

At angle θ $M_\theta + F(R-x) = 0$



$$M_\theta = -F(R-x)$$
$$= -F(R - R\cos\theta)$$
$$= -FR + FR\cos\theta$$
$$M_\theta = R[F\cos\theta - F]$$

$$U_{AB} = \int_0^L \frac{M_\theta^2}{2EI}\,ds \qquad ds = R\,d\theta$$

$$= \int_0^\pi \frac{M_\theta^2}{2EI} R\,d\theta$$

$$= \int_0^\pi \frac{R^2[F\cos\theta - F]^2 R}{2EI}\,d\theta$$

$$= \int_0^\pi \frac{R^3}{2EI}\left[F^2 + F^2\cos^2\theta - 2F^2\cos\theta\right]d\theta$$

$$= \int_0^\pi \frac{R^3F^2}{2EI}\left[1 + \cos^2\theta - 2\cos\theta\right]d\theta \qquad \cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$= \frac{R^3F^2}{2EI}\int_0^\pi \left[1 - 2\cos\theta + \frac{1+\cos 2\theta}{2}\right]d\theta$$

$$= \frac{R^3F^2}{2EI}\int_0^\pi \left[\frac{3}{2} - 2\cos\theta + 1 + \frac{\cos 2\theta}{2}\right]d\theta$$

(ii)

MM2 MS2
0910

Q4 (cont)

$$= \frac{R^3F^2}{4EI}\int_0^{\pi}\left[3 - 4\cos\theta + \cos 2\theta\right] d\theta$$

$$= \frac{R^3F^2}{4EI}\left[3\theta - 4\sin\theta + \frac{\sin 2\theta}{2}\right]_0^{\pi}$$

$$= \frac{R^3F^2}{4EI}\left[3\pi\right]$$

$$U_{AB} = \frac{3\pi R^3F^2}{4EI}$$

$$M_{BC} = F.2R$$

$$U_{BC} = \int_0^L \frac{M_{BC}^2}{2EI}\,ds$$

$$= \int_0^L \frac{4F^2R^2}{2EI}\,ds$$

$$= \frac{4F^2R^2L}{2EI}$$

$$U_{TOT} = U_{AB} + U_{BC} = \frac{3\pi R^3F^2}{2EI} + \frac{4F^2R^2L}{2EI}$$

$$= \frac{R^2F^2}{2EI}\left[\frac{3\pi R}{2} + 4L\right]$$

$$u_F = \frac{\partial U_{TOT}}{\partial F} = \frac{R^2F}{EI}\left[\frac{3\pi R}{2} + 4L\right]$$

$$E = 209.10^9 \ \mathrm{N/m^2}$$

$$I = \frac{\pi}{64}\left[D_o^4 - D_i^4\right]$$

$$= \frac{\pi}{64}\left[0.1^4 - 0.094^4\right]$$

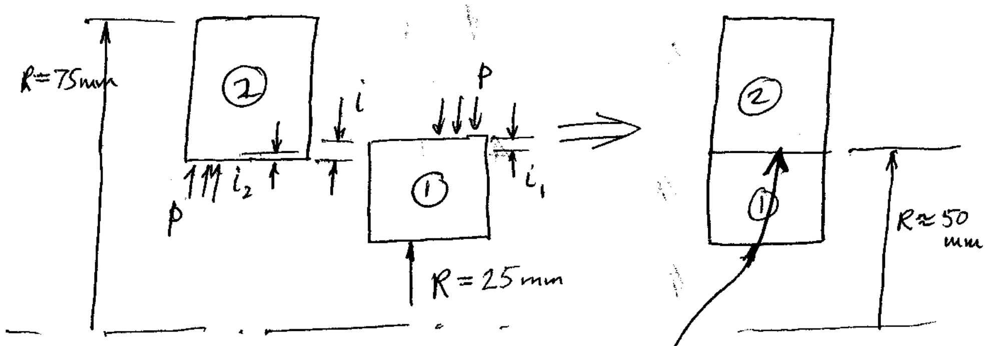
$$= \underline{1.076 . 10^{-6}\ \mathrm{m}^4}$$

$$= \frac{1.5^2 . 250}{209.10^9 . 1.076 . 10^{-6}}\left[\frac{3\pi . 1.5}{2} + 4.6\right]$$

= [redacted] $\underline{77.7\ \mathrm{mm}}$

[20 marks]

Q.5



Lame's equations:- $\sigma_r = A - \frac{B}{r^2}$, $\sigma_\theta = A + \frac{B}{r^2}$

$\sigma_\theta = -100$ MPa.
$E = 200 \times 10^3$ MPa
$\nu = 0.3$

The radial interference, i, is the sum of the radial deformations, i_1 and i_2, i.e.,

$$i = i_1 + i_2 \qquad (1).$$

Cylinder ①

$$\sigma_\theta = A_1 + \frac{B_1}{r^2}$$

$$\sigma_r = A_1 - \frac{B_1}{r^2}$$

At $r = 25$ mm, $\sigma_r = 0$,

$$\therefore \quad 0 = A_1 - \frac{B_1}{25^2} \qquad (2)$$

At $r = 50$ mm, $\sigma_r = -p$ (the interface pressure).

$$\therefore \; -p = A_1 - \frac{B_1}{50^2} \qquad (3)$$

Q5 cont

from equation (2) $B_1 = 25^2 A_1$

Substitute for B_1 into equation (3) gives:-

$$-p = A_1 = \frac{25^2 A_1}{50^2} = A_1 - \frac{A_1}{4} = \frac{3A_1}{4}$$

$$\therefore A_1 = -\frac{4}{3}p \qquad (4)$$

Substitute for A_1 into (2) gives:-

$$-\frac{4}{3}p - \frac{B_1}{25^2} = 0$$

$$\therefore B_1 = -25^2 \times \frac{4}{3}p \qquad (5)$$

At $r = \cancel{X}\ 50$ mm, $\sigma_\theta = -100$ MPa.

$$\therefore -100 = -\frac{4}{3}p - \cancel{25^2} \times \frac{4}{3}\frac{p}{\cancel{50^2}\,4} = -\frac{5}{3}p.$$

$$\therefore p = \frac{3}{5} \times 100 \text{ MPa}$$

ie p = [redacted] MPa (6) [10 marks]

$$\varepsilon_\theta = \frac{u}{r} = \frac{1}{E}(\sigma_\theta - \nu\sigma_r) \qquad (7) \text{ assuming } \sigma_z = 0$$

∴ For cylinder 1 at the interface.

$$-\frac{i_1}{50} = \frac{1}{E}(\sigma_\theta - \nu\sigma_r) \qquad (8)$$

where $\sigma_\theta = A_1 + \frac{B_1}{50^2} = -\frac{4}{3}p - 25^2 \times \frac{4}{3}\frac{p}{50^2}$

$$= -\frac{4}{3}p\left(1 + \frac{1}{4}\right) = -\frac{5p}{3} = -100 \text{ MPa}$$

Q5 cont

i.e. $\sigma_\theta =$ [redacted] -100 MPa

and $\sigma_r =$ [redacted] -60 MPa.

Substituting for σ_θ and σ_r in (8) gives

$$-\frac{i_1}{50} = \frac{1}{200 \times 10^3}\left(\text{[redacted]}\,{-100} - 0.3 \times (\text{[redacted]}\,{-60})\right)$$

i.e. $i_1 =$ [redacted] 0.0205 mm. (9)

0.012813

Cylinder (2)

$$\sigma_r = A_2 - \frac{B_2}{r^2}$$

At $r = 75$ mm, $\sigma_r = 0$

$$\therefore \quad 0 = A_2 - \frac{B_2}{75^2}$$

i.e. $\underline{B_2 = 75^2 A_2}$ (10)

At $r = 50$ mm, $\sigma_r = -p$

i.e. $-p = A_2 - \dfrac{B_2}{50^2}$ (11)

Substitute (10) into (11) gives:-

$$-p = A_2 - \frac{75^2 A_2}{50^2} = A_2 - \frac{9}{4}A_2 = -\frac{5}{4}A_2$$

$$\therefore \quad \underline{A_2 = \frac{4p}{5}}$$

Hence, $\sigma_r = \dfrac{4p}{5} - \dfrac{75^2 \times 4p}{5r^2} = \dfrac{4p}{5}\left(1 - \dfrac{75^2}{r^2}\right)$

At $r = 50$ mm $\quad \sigma_r = \dfrac{4p}{5}\left(1 - \dfrac{9}{4}\right) = -p$ ✓

Q5 cont

and $\sigma_\theta = \frac{4p}{5} + 75^2 \times \frac{4p}{5\,r^2}$

ie $\sigma_\theta = \frac{4p}{5}\left(1 + \frac{75^2}{r^2}\right)$

At $r = 50$ mm, $\sigma_\theta = \frac{4p}{5}\left(1 + \frac{9}{4}\right)$

156 MPa

$= \frac{4}{5} \times \frac{13}{4} p = \frac{13}{5} p =$ [redacted]

$\therefore \frac{i_2}{50} = \frac{1}{200 \times 10^3}$ (156 [redacted] $- 0.3 \times (-$ 60 [redacted] $))$ $\frac{\cancel{MPa}}{\cancel{MPa}}$

0.0435

$\therefore i_2 =$ [redacted] mm

$\therefore$ Diametral interference $= 2 \times (i_1 + i_2)$

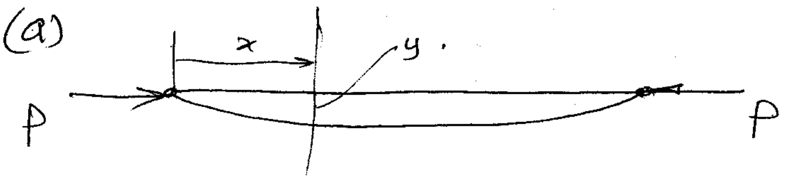
$= 2 \times ($0.0205 [redacted] $+$ 0.0435 [redacted]$)$

0.128

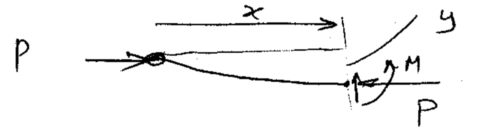
$=$ [redacted] mm. [10 marks]

0.128 mm.

Q6 (a)



FBD of LH section



Equilibrium (Moments)

$$M = Py$$

$$EI\frac{d^2y}{dx^2} = -M$$

$$\therefore \quad EI\frac{d^2y}{dx^2} = -Py$$

ie $$\frac{d^2y}{dx^2} + \frac{P}{EI}y = 0$$

Let $\frac{P}{EI} = \alpha^2$, then $\frac{d^2y}{dx^2} + \alpha^2 y = 0$.

Solution:- $y = A\sin(\alpha x) + B\cos(\alpha x)$

Boundary conditions:-

At $x=0$, $y=0$ $\therefore$ $B=0$

At $x=l$, $y=0$ $\therefore$ $0 = B\cos(\alpha l)$

$\therefore B=0$ (trivial solution) or $\cos(\alpha l)=0$.

Q6 cont

Hence $\alpha l = n\pi$

i.e. $\alpha^2 l^2 = n^2\pi^2$

$$\frac{P}{EI} l^2 = n^2\pi^2$$

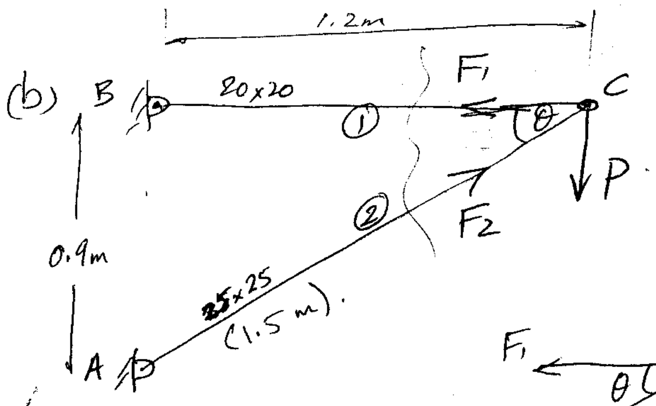
i.e. $P = n^2\pi^2 \dfrac{EI}{l^2}$ [8 Marks]

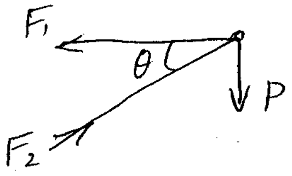
The lowest buckling load is when $n=1$
($n=0$ is a trivial solution)

Hence, the buckling load, P_c, is given by:-

$$P_c = \pi^2 \frac{EI}{l^2}$$

(b)





$AC = \sqrt{0.9^2 + 1.2^2} = 1.5$

~~AC = ...~~

$\therefore \sin\theta = \dfrac{3}{5}$ and $\cos\theta = \dfrac{4}{5}$

Q6 cont

Vertical equilibrium

$$P = \sin(\theta) \times F_2$$

$$\therefore \; F_2 = \frac{5P}{3}.$$

Horizontal equlibrium

$$F_1 = F_2 \cos\theta \; = \; \frac{5P}{3} \times \frac{4}{5}.$$

$$\therefore \; F_1 = \frac{4P}{3}$$

Bar (1) is in tension and may fail due to plastic collapse, ie when

$$\sigma_y A_1 = F_1 \quad = \frac{4P}{3}$$

ie $\quad P = \frac{3}{4} \times 100 \frac{N}{mm^2} \times 20\,mm \times 20\,mm$

$\quad = 30{,}000N$

ie $P = 30kN$

Bar (2) is in compression and so may fail due to plastic collapse or buckling.

Plastic collapse :— $\quad \sigma_y A_2 = F_2$

ie $\sigma_y \times 25 \times 25 = \frac{5P}{3}$

ie $P = \frac{3}{5} \times 100 \times 25 \times 25 \quad N = 37500\,N$

Q6 cont

ie $\underline{P = 37.5 \text{ kN}}$

<u>buckling:-</u>

$$P = \frac{\pi^2 EI}{\ell^2}$$

$$= \frac{\pi^2 \times 200 \times 10^3 \text{ N/mm}^2}{1500^2 \text{ mm}^2} \times \frac{25^4 \text{ mm}^4}{12}$$

$$= 28,56 ~~5.7~~$$

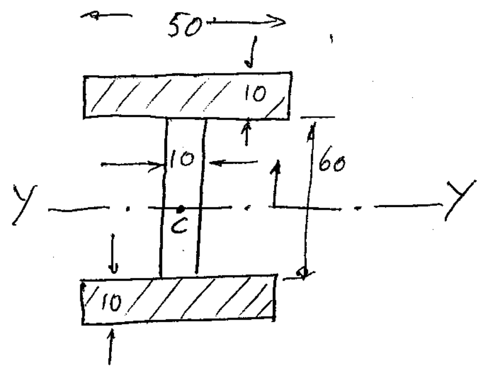
ie $\underline{P = 28.6 \text{ kN}}$

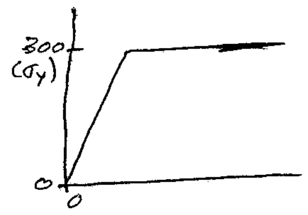
The lowest value of P is the value associated with the buckling of bar AC,

<u>ie maximum allowable load is</u>

<u>28.6 kN</u> [12 Marks]

Q.7





Yielding occurs through the whole of the flanges, ∴

$30 < y < 40$, $\sigma = 300$ MPa.

and $-30 > y > -40$, $\sigma = -300$ MPa.

for $-30 < y < 30$, $\sigma = 300 \frac{y}{30} = 10y$

Moment equilibrium gives :-

$$M = \int_A \sigma \times y \times dA$$

ie $$M = 2\left\{ \int_0^{30} 10y \times y \times 10\,dy + \int_{30}^{40} 300 \times y \times 50\,dy \right\}$$

$$= 2\left\{ \int_0^{30} 100y^2\,dy + \int_{30}^{40} 15000\,y\,dy \right\}.$$

$$= 200\left\{ \left[\frac{y^3}{3}\right]_0^{30} + \left[150\frac{y^2}{2}\right]_{30}^{40} \right\} \quad \text{N mm}$$

$$= 200\left\{ 9000 + 75(1600 - 900) \right\}$$

$$= 12.3 \times 10^6 \text{ N mm}$$

ie $\underline{M = 12.3 \text{ kN m}}$ [10 Marks]

Q7 cont.

Compatability (plane sections remain plane) gives:-

$$\varepsilon = \frac{y}{R}$$

At $y = 30$, $\sigma = \sigma_y = 300 \frac{N}{mm^2}$ and is just at the point of going plastic, but it is still elastic, therefore

$$\varepsilon = \frac{\sigma_y}{E} = \frac{300\ N/mm^2}{200 \times 10^3\ N/mm^2} = 1.5 \times 10^{-3}$$

ie $\varepsilon = \frac{y}{R}$ leads to $1.5 \times 10^{-3} = \frac{30}{R}$.

ie $R = 30 \div 1.5 \times 10^{-3} = 20 \times 10^3 mm$.

$\therefore$ $\underline{R = 20 m}$

Assuming that unloading is elastic we can use :-

$$\frac{(\Delta\sigma)}{y} = \frac{(\Delta M)}{I}$$

$$\therefore (\Delta\sigma) = \frac{12.3 \times 10^6\ Nmm}{I} \times y$$

$$I = \left(\frac{50 \times 80^3}{12} - \frac{40 \times 60^3}{12} \right) mm^4$$

$$= (2.13\dot{3} \times 10^6 - 0.72 \times 10^6)\ mm^4$$

$$\underline{I = 1.41\dot{3} \times 10^6\ mm^4}$$

$$\therefore \Delta\sigma = \frac{12.3 \times 10^6\ Nmm}{1.41\dot{3} \times 10^6\ mm^4}\ y = \underline{8.703 y}$$

Q7 cont

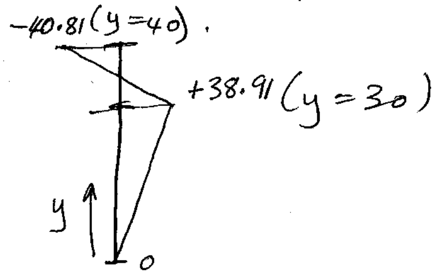
At $y = 30$, $\Delta\sigma = 261.09$ and $\therefore \sigma = 300 - 261.09$

$= 38.91 \ \frac{N}{mm^2}$

At $y = 40$, $\Delta\sigma = 348.12$ and $\therefore \sigma = 300 - 348.12$

$= -48.12 \ \frac{N}{mm^2}$

Residual stress distribution



At $y = 30$ the stress is at the limit of elastic behaviour,

$$\therefore \frac{30\,mm}{R} = \frac{38.91\ N/mm^2}{200 \times 10^3\ N/mm^2}$$

$$\therefore R = 1.542 \times 10^5\ mm$$

ie $R_{residual} = 154.2\ m$ [10 Marks]

8(a)

8(b)

See lecture notes.

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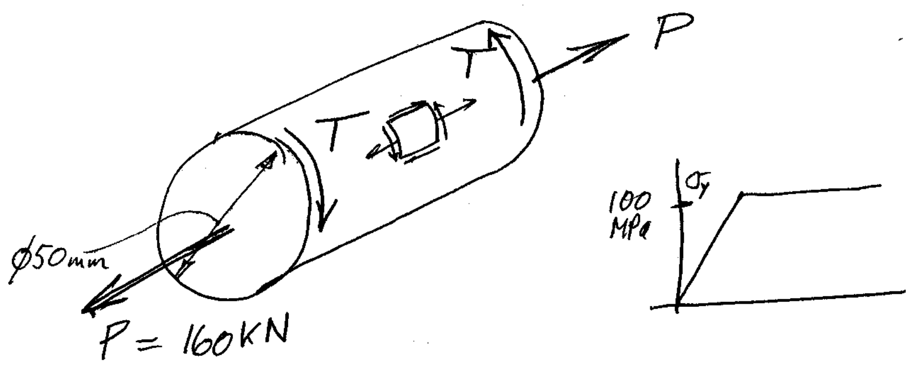
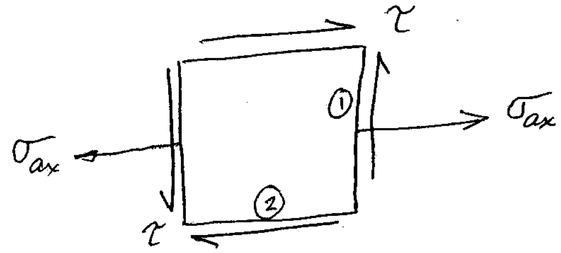
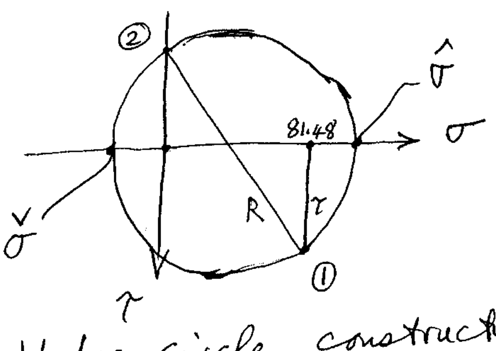
cont (C)





$$\sigma_{ax} = \frac{160 \times 10^3 \ N}{\pi \times 25^2 \ mm^2} = \underline{81.48 \ \frac{N}{mm^2}}$$

Mohrs circle



From the Mohrs circle construction, it is clear the $\hat{\sigma}$ and $\check{\sigma}$ are the maximum and minimum principal stresses; on is +ve and the other is −ve and hence the third principal

Q8(c)

stress is intermediate ($\sigma_3 = 0$), ie the Tresca yield criterion applied to this case gives :-

$$\hat{\sigma} - \check{\sigma} = \sigma_y$$

From Mohr's circle, $R = \sqrt{\left(\frac{81.48}{2}\right)^2 + \tau^2}$

and $\hat{\sigma} = \frac{81.48}{2} + R$

$\check{\sigma} = \frac{81.48}{2} - R$.

$\therefore$ Tresca's yield criterion gives

$$\hat{\sigma} - \check{\sigma} = \sigma_y$$

ie $\quad 2R = \sigma_y$

$$\therefore 2\sqrt{\left(\frac{81.48}{2}\right)^2 + \tau^2} = 100$$

$$1660 + \tau^2 = 50^2$$

$$\text{ie } \tau = 28.98 \frac{N}{mm^2}$$

In the elastic range,

$$\frac{T}{J} = \frac{\tau}{r} \qquad \therefore T = \frac{\pi \times 50^4}{32} mm^4 \times \frac{28.98 \frac{N}{mm^2}}{25 mm}$$

$$\therefore T = 7.11 \times 10^5 \ Nmm$$

[8 Marks]